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On the Interaction of Elementary Particles. I.

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§1. Introduction

At the present stage of the quantum theory little is known about the nature of interaction between elementary particles. For example, the ~~force acting~~ interaction ~~We are not sure~~ whether between a neutron and a proton is an ordinary attraction ~~force~~ or an "Platzwechsel" ~~exchange interaction~~ first proposed by Heisenberg. Recently Fermi[1] has treated the problem of β-ray disintegration on the hypothesis of ~~the existence of~~ "neutrino"[*]. According to this theory a neutron and a proton can interact by emitting and absorbing a neutrino and an electron. Unfortunately the energy of interaction calculated on ~~this~~ such assumption[2] is much too small to account for the binding of neutrons and protons in the nucleus. To remove this defect we ~~can't~~ may have to modify the theory of Heisenberg or Fermi in the following way.

The transition of a heavy particle from a neutron state to a proton state is not always accompanied

(1) E. Fermi, Zeits. f. Phys. 88, 161, (1934).

(2) Ig. Tamm, Nature, 133, 981 (1934); D. Iwanenko, ibid. 981 (1934).

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by the emission of ~~a neutr~~ the light particles, i.e. a~~n~~ neutrino and an electron, but the energy liberated by the transition can sometimes be taken up by another heavy particle, which ~~changes~~ transfers into a neutron state.

in a proton state,) Consideration of such a process ~~the~~

between a neutron and ~~to~~ a proton.
~~interaction increases~~ will increase ~~much~~ much the interaction by ~~of~~ exchange of energy, [

Now such ~~direct interaction~~ between the elementary particles can be described by means of the aid of a field of force just as the interaction between charged particles ~~can~~ are described by ~~electrons~~ means of the aid of electromagnetic field.

~~the author~~ will ~~first~~ discuss briefly
In this paper the nature of such a field ~~of force~~ and of the quantum, which accompanies ~~the field~~ ~~it~~ the field, just as the photon accompanies the electromagnetic field, ~~when it is quantized~~. Also their bearing on the ~~theory~~ ~~of~~ nuclear structure, ~~and of~~ the β-ray disintegration and the scattering of neutrons or neutrinos will be ~~discussed~~ considered.

Fuller account will be made in the next paper.

§ 2. Field describing the ~~exchange~~ interaction ~~interaction~~ interaction of ~~elementary~~ part.

In analogy to ~~electr~~ scalar ~~and~~ ~~the~~ vector potentials V, A of the ~~electromagnetic~~ electromagnetic field we introduce a scalar function $U(x,y,z,t)$ and a vector function $\mathbf{B}(x,y,z,t)$ describing the field between a neutron and a proton. These functions ~~will~~ may will satisfy ~~the wave~~ equations similar to the wave equations for the electromagnetic potentials. ~~But now!~~

Now the equation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\}U = 0 \qquad (1)$$

has only static solution with ~~of~~ central symmetry

$$\frac{1}{r}$$

except the additive and multiplicative constants ~~and multiplicative constants~~,

The potential of the ~~force between~~ acting between a neutron and a proton, however, is recognized ~~to~~ not to be Coulomb type, but to decrease more rapidly with the distance. ~~which~~ It can be expressed for example by

$$g^2\frac{e^{-\lambda r}}{r}, \qquad (2)$$

where g is a constant corresponding to the electric charge in ~~the~~ case of electromagnetic field. Since this function is, as well known, the static solution with central symmetry of the wave equation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\}U = 0, \qquad (3)$$

† The equations (3) and (4) can be deduced ~~by the~~ variation of $U, U^\dagger$ from a Lagrangian function

$$L_F = \iiiint \left\{ \frac{\partial U^\dagger}{\partial x}\frac{\partial U}{\partial x} + \frac{\partial U^\dagger}{\partial y}\frac{\partial U}{\partial y} + \frac{\partial U^\dagger}{\partial z}\frac{\partial U}{\partial z} - \frac{1}{c^2}\frac{\partial U^\dagger}{\partial t}\frac{\partial U}{\partial t} \pm \lambda^2 U^\dagger U \pm g(\tilde{\Psi} U^\dagger \Psi + \tilde{\Psi} U \Psi) \right\} dV\,dt,$$

~~where~~ by the variation of $U, U^\dagger$, ~~where~~ their values on the three dimensional boundary being fixed.

Next we consider the ~~complex conjugate field $U^\dagger$~~ function $U^\dagger$ complex conjugate to U, ~~which~~ satisfying the equation

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right) \tilde{U}^{\dagger} = -g \tilde{\Psi} \Psi. \qquad (4)$$

This corresponds to the inverse transition from a proton state to a neutron state.
~~vector potential~~ Similar equations will hold for the vector potential, but we ~~do not consider~~ disregard it for the moment, because we do not know the ~~correct~~ relativistic wave equations for the neutron and the proton.

let us assume (2) to be the correct wave equation for U in vacuum, where λ is a constant with dimension cm^{-1}. In the presence of the heavy particles the U-field interact with them and causes the transition from ~~the~~ neutron state to ~~the~~ proton state, so that, if we denote the wave functions of ~~the~~ neutron and ~~the~~ proton by $\Psi(xyzt)$ and $\Phi(xyzt)$ respectively, in the ~~right hand side of (2) can be substituted by~~ the ~~ter density term like~~ wave equation becomes

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\} U = \cancel{-g\tilde{\Psi}\Phi} \quad 4\pi g \tilde{\Phi}\Psi \qquad (4)$$

~~should be inserted in the right hand side of the equation (2).~~

complex conjugate

Next we consider ~~the~~ field $U^{\dagger}$, which is the complex conjugate ~~of~~ to U. This corresponds to the inverse transition from a proton state to a neutron state, ~~and~~ ~~$U^{\dagger}$~~ satisfies the equation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\} U^{\dagger} = -g\tilde{\Psi}\Phi. \qquad (4)^{\dagger}$$

Now ~~the wave~~ equations for heavy particles interacting with U-field, can be deduced from a ~~Hamiltonian~~ Lagrangian function for the total system

$\tilde{\Psi}\, i\hbar \frac{\partial \Psi}{\partial t} + \tilde{\Phi}\, i\hbar \frac{\partial \Phi}{\partial t}$

The kinetic and potential energies of the heavy particles are

$$\frac{\hbar^2}{2M_N}\iiint\left(\frac{\partial\tilde{\Phi}}{\partial x}\frac{\partial\Phi}{\partial x}+\frac{\partial\tilde{\Phi}}{\partial y}\frac{\partial\Phi}{\partial y}+\frac{\partial\tilde{\Phi}}{\partial z}\frac{\partial\Phi}{\partial z}\right)dv$$

$$+\frac{\hbar^2}{2M_P}\iiint\left(\frac{\partial\tilde{\Psi}}{\partial x}\frac{\partial\Psi}{\partial x}+\quad\right)dv$$

and

$$g\iiint(\tilde{\Phi}U\Psi+\tilde{\Psi}U^{+}\Phi)\,dv,$$

so that the ~~Lagrangian for function~~ the equation of motion can be derive from ~~wave~~ for the heavy particle

~~$\Psi \frac{1}{2M_N}$~~

$$L_M=\iiiint\left\{\frac{\hbar^2}{2M_N}(\quad)+\frac{\hbar^2}{2M_P}(\quad)\right.$$

$$\left.-g(\tilde{\Phi}U\Psi+\tilde{\Psi}U^{+}\Phi)\right\}dv\,dt.$$

by the variation of $\Psi\tilde{\Psi}, \Phi, \tilde{\Phi}$, i.e.

$$\left(\frac{\hbar^2}{2M_N}\Delta+i\hbar\frac{\partial}{\partial t}\right)\Phi=-gU\Psi$$

$$\left(\frac{\hbar^2}{2M_P}\Delta+i\hbar\frac{\partial}{\partial t}\right)\Psi=-gU^{+}\Phi$$

Now the ~~two~~ constants g, λ appearing in the equations (4), (5) should be determined by comparing with experi~~ence~~ment. For example, assuming the "Platzwechsel" interaction between a neutron and a proton, (2) for the mass defect of H^2 and the scattering probability of a neutron by a proton can be ~~estimated~~, ~~which from~~ which calculated and compared with experimental data.

Thus ~~it is estimated that~~ ~~λ should be a few multiple~~ λ is estimated to lie between 10^{12} cm and 10^{13} cm and g ~~should be~~ to be a few multiple of elementary charge e.

§ 3. Quantum accompanying the U-field

~~Now~~ In the quantum theory the U-field should be quantized according to the general principle.

~~In free space U~~ Denoting

$$p_x = -i\hbar \frac{\partial}{\partial x}, \quad \text{etc} \qquad W = i\hbar \frac{\partial}{\partial t}$$

$$m_U = \lambda \hbar,$$

the wave equation for U in free space can be ~~is~~ written in the form

$$\{ p_x^2 - W^2/c^2 + m_U^2 c^2 \} U = 0,$$

so that ~~we see~~ the quantum accompanying the U-field has ~~an~~ proper mass $m_U = \frac{\lambda\hbar}{c}$.

Assuming, for example, $\lambda = 5 \times 10^{12}$ cm

we obtain for m_U a value $2 \times 10^2 \times$ electron mass.

Thus the proper mass of the quantum is very great and lie between the masses of the electron and the proton.
~~If such~~ Now we consider ~~a massive quantum~~ in what case such a massive quantum will be emitted from the nucleus? ~~The energies of~~ If the energies of the neutron state Φ and the proton state Ψ be W_N, W_P respectively, we can write

$$\Phi = u(xyz)\, e^{-iW_N t/\hbar}$$

$$\tilde{\Psi} = \tilde{v}(xyz)\, e^{iW_P t/\hbar},$$

~~so that the~~ Hence the right hand side of the ~~wave~~ equation (4) becomes

$$-4\pi g \cdot \tilde{v} u\, e^{-i(W_N - W_P)t/\hbar},$$

so that putting

$$U = U'(xyz)\, e^{\cancel{-i(W_N-W_P)}-i\omega t},$$

we have

$$\left\{ \Delta - \left(\lambda^2 - \frac{\omega^2}{c^2}\right) \right\} U' = -4\pi g\, \tilde{v} u,$$

where $\omega = W_N - W_P / \hbar$,
Integrating this equation we obtain a solution

$$U(r) = g \iiint \frac{e^{-\mu |r - r'|}}{|r - r'|}\, \tilde{v}(r')\, u(r')\, dv'$$

where $\mu = \sqrt{\lambda^2 - \frac{\omega^2}{c^2}}$, In case $\lambda > \frac{W}{c}$ or $m_U c^2 > W_N - W_P$, μ is real and the function $J(r)$ of Heisenberg ~~or~~ has the form $e^{-\mu r}/r$, but μ ~~is not~~ depends on ~~the~~ $W_N - W_P$ ~~and~~ and becomes smaller and smaller ~~and smaller~~ when the latter becomes nearer and nearer to $m_U c^2$. This means ~~the~~ range of interaction between a neutron and a proton ~~extends~~ ~~bec~~ increases when ~~the~~ $W_N - W_P$ increases.

According to the experiment of Bonner[3] the collision effective cross section of ~~scattering~~ the neutron and ~~fro~~ the nucleus ~~de~~ de-increases with the velocity of the neutron in case of ~~Pb~~ lead ~~and~~, whereas it decrease in case of carbon and hydrogen. Moreover the rate of decrease is slower in case of carbon than hydrogen. This effect is not clear ~~may~~ and may be due to the superposition of inelastic scattering and ~~on~~ the simple elastic scattering in case of the heavy nucleus. Above consideration show, however, a possible explanation of the effect. ~~for ex.~~ That is, for, if the binding energy of the proton to the nucleus becomes comparable to $m_U c^2$, the interaction of the neutron and the proton will ~~change~~ increase considerably with the velocity of the neutron, so that the cross section will decrease slower in this case than in the case of hydrogen. Now the binding energy of the outer proton of C^{12} is ~~determ~~ estimated

(3) T. W. Bonner, Phys. Rev. 45, 606 (1934).

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from the difference of masses of C^{12} and B^{11}, which is

$$12.0036 - 11.0110 = 0.9926.$$

This corresponds to a binding energy

$$0.0152$$

in mass unit and ~~th~~ about thirty times of the electron mass, so that in case of carbon we can expect the effect observed by Bonner. Of course the above argument is only tentative and the other explanations are not excluded.

Next in case $\lambda < \frac{w}{c}$ or $m_U c^2 < W_N - W_P$, μ becomes imaginary and U expresses a spherical wave, so that ~~in~~ a quantum is emitted with energy greater than $m_U c^2$ is emitted by ~~such~~ the transition of a neutron state to a proton state in outer space, provided the heavy particle from $W_N - W_P) \gg m_U c^2$. The phase velocity of U-wave is greater than ~~the~~ light velocity c, but the group velocity is smaller than ~~it~~ c, as in the case of electron wave.

The mass m_U is, however, hundred times greater than electron mass, so that the condition $W_N - W_P > m_U c^2$ is ~~rarely~~ never satisfied in ordinary case.

The quantum accompanying U ~~has~~ should moreover have a ~~positive~~ negative charge $-e$ ~~and accordin~~ on account of the conservation of charge and ~~also~~ ~~follow~~ satisfy the Bose's statistics, since both ~~the~~ neutrons and ~~the~~ protons ~~follow~~ satisfy ~~to~~ Fermi's statistics.

So ~~that the~~ the quantization of U-field will be performed analogous to electromagnetic field. ~~but~~ but in ~~in~~ ~~this~~ the present paper we do not go farther ~~in this problem~~.

The detailed account of the problem will be given in the

§ 4. Theory of β-Disintegration

According to our theory the ~~U-field~~ massive quantum emitted ~~by a heavy particle~~ when a heavy particle jumps from a neutron state to a proton state can be absorbed by a light particle which jumps from a neutrino state of negative energy to an electron state of positive energy.

Thus an antineutrino and an electron are emitted from the nucleus. Such ~~double processes~~ intervention of a massive quantum does not alter ~~the transition~~ probability of β-disintegration ~~essentially~~ in general, for which ~~the~~ has been calculated on the hypothesis of direct coupling of a heavy particle and a light particle, just as in the theory of internal conversion the intervention of the ~~γ-ray~~ photon does not alter ~~the~~ in general coefficient of internal conversion(4).

(4) Taylor and Mott, Proc. Roy. Soc. A. 138, 665 (1932).

so that our theory ~~differ~~ does not differ essentially from Fermi's theory.

Fermi has assumed that an electron and a neutrino are emitted simultaneously from the radioactive nucleus, but ~~we as want~~ this assumption is formally for the sake of simplicity in form ~~we~~ ~~assume that a light particle~~ equivalent to assume that a light particle jumps from a neutrino state of negative energy ~~at~~ to a electron state of positive energy.

For if the eigenfunctions of the electron and the neutrino be ψ_k, φ_k respectively, where $k = 1, 2, 3, 4$, in the right hand side of the equation for U a term of the form

$$-4\pi g' \sum_{k=1}^{4} \tilde{\psi}_k \varphi_k \qquad (6)$$

will be added, where g' is ~~an~~ a constant with the same dimension as g. Now ~~if φ_k' be~~ the eigenfns of the neutrino state with ~~the same energy~~ energy and ~~momentum~~ just opposite to ~~the state~~ that of the state φ_k is given by

$$\varphi_k' = \delta_{kl} \tilde{\varphi}_l$$

where

$$\delta = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix},$$

so that (6) becomes

$$-4\pi g' \sum_{k=1}^{4} \tilde{\psi}_k \delta_{kl} \tilde{\varphi}_k \qquad (7)\ (\cancel{7})$$

This expression of density is the, ~~conjugate complex to~~ same as ~~same~~ as that of Fermi ~~except the conjugate complex and their~~ The absolute ~~values~~ value ~~are~~ is the same ~~in both case~~.

Now from (4) and (6) we obtain for the matrix element of the interaction energy of ~~the~~ the heavy particle and the light particle

$$g g' \iint \tilde{v}(r_1)\, u(r_1) \sum_k \tilde{\psi}_k(r_2)\, \varphi_k(r_2) \times \frac{e^{-\lambda r_{12}}}{r_{12}}\, dv_1\, dv_2, \qquad (8)$$

from which we can calculate ~~the~~ the probability of β-disintegration.

Since λ is ~~far~~ very large ~~compared~~ compared with the wave numbers of the electron and the neutrino in ordinary case, the function

$$\frac{e^{-\lambda r_{12}}}{r_{12}}$$

for the integration with respect to $x_2 y_2 z_2$

can be regard as a δ-function multiplied by a factor $\frac{4\pi}{\lambda^2}$, for

$$\int \frac{e^{-\lambda r_{12}}}{r_{12}}\, dv_2 = \frac{4\pi}{\lambda^2},$$

so that (8) becomes

$$\frac{4\pi g g'}{\lambda^2}\iiint \tilde{v}(x)\,u(x)\sum_k \tilde{\psi}_k(x)\,\varphi_k(x)\,.\,dv \qquad (9)$$

~~which~~, considerin ~~The absolute~~ the square of the ~~modulus of~~ Since this expression is the same as the ~~equation~~ equation (21) of Fermi ~~, so that except~~, if we substitute for Fermi's g the fector

$$\frac{4\pi g g'}{\lambda^2},$$

the our theory does not differ essentially from Fermi's theory. Comparing with experimental data, Fermi has taken for his ~~g~~ g a value $4\cdot 10^{-50}$ cm^3.erg, from which we can determine g' ~~by~~. If ~~we take for~~ we take for λ a value $\approx 5\times 10^{12}$ and for $g \approx 3\times 10^{-9}$, ~~g' becomes a~~

$$g' = \frac{\lambda^2}{4\pi g}\,4\cdot 10^{-50}$$

$$= 3\times 10^{-17},$$

which is 10^8 times smaller than g. This means that the interaction between a neutrino and a electron is much smaller than that between a neutron and a proton, so that the neutrino ~~is~~ will be far more penetrating than the neutron and more difficult to observe. The difference of g and g' ~~will~~ may be due to the difference of masses of heavy and light particles.

§5. Conclusion

~~In conclusion~~ Thus the interaction of elementary particles can be described by considering ~~a~~ the quanta with elementary charge and proper mass far larger than that of the electron, satisfying the Bose's statistics. The interaction of these quanta with heavy particles ~~is far great~~ should be far greater than that with light particles to account for the large interaction of a neutron and a proton and the small probability of β-disintegration. ~~The~~ ~~The proper mass of these quanta should be surprisingly large to account for the interaction the or~~

Such quanta, if they ~~emitted from a nucleus~~ come nearer to the matter ~~the nucleus~~, ~~they~~ will give~~s~~ ~~their~~ charge and energy to ~~the~~ it, ~~nuclei and~~, ~~the transm~~ charge of ~~the~~ matter which ~~changes~~ So that if the quanta with negative charge come in excess, the matter will be charged up to negative potential and vice versa.

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character

~~but~~ These arguments ~~merely~~, of course, of merely speculative, agree with the ~~charged~~ view ~~recently~~ that the ~~cosmic~~ high speed positive particles ~~consisting~~ in the cosmic ray are generated by ~~the earth's~~ electrostatic field[(5)]. of the earth, which are charged to a negative potential.

(5) L. G. H. Huxley, Nature, 134, 418, 571 (1934); Johnson, Phys. Rev. ~~45~~, 569 (1934).